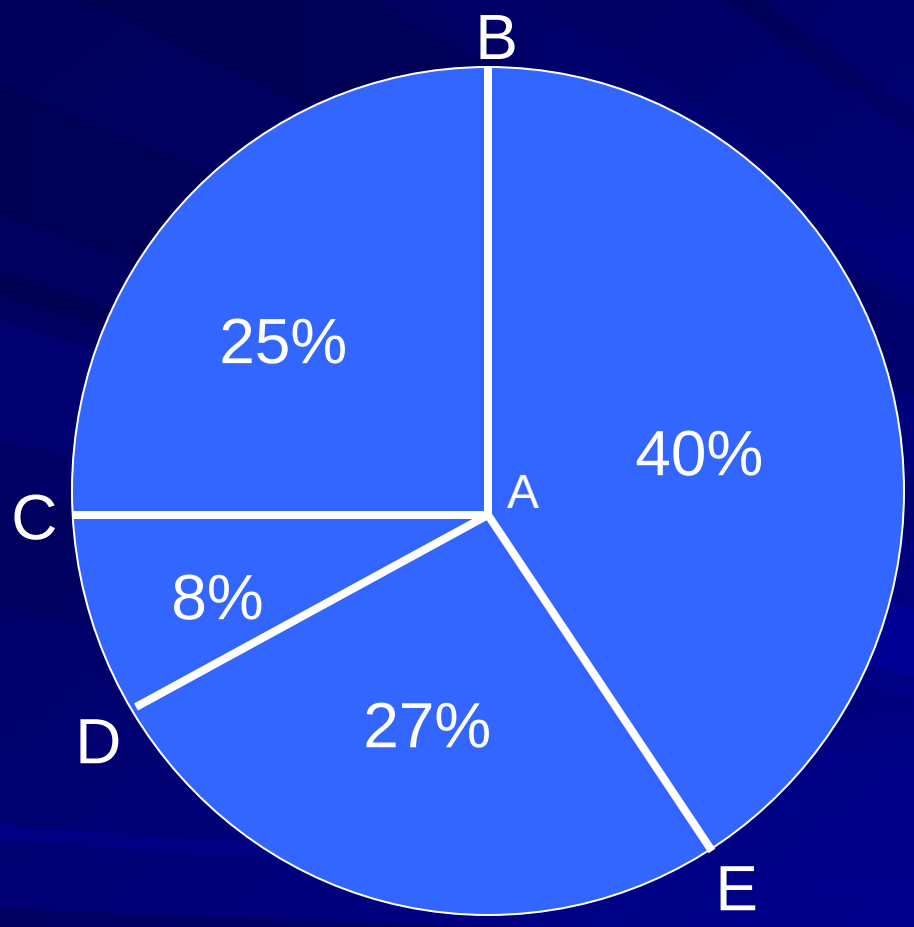
Circles and Arcs
Day 2

Finding Arc Measure and Arc Length

Finding measures of Central ∠s



$m\angle BAE = 40\%$ of 360

$m\angle BAE = (.40) \cdot 360$

$m\angle BAE = 144°$

$m\angle CAD = 8\%$ of 360

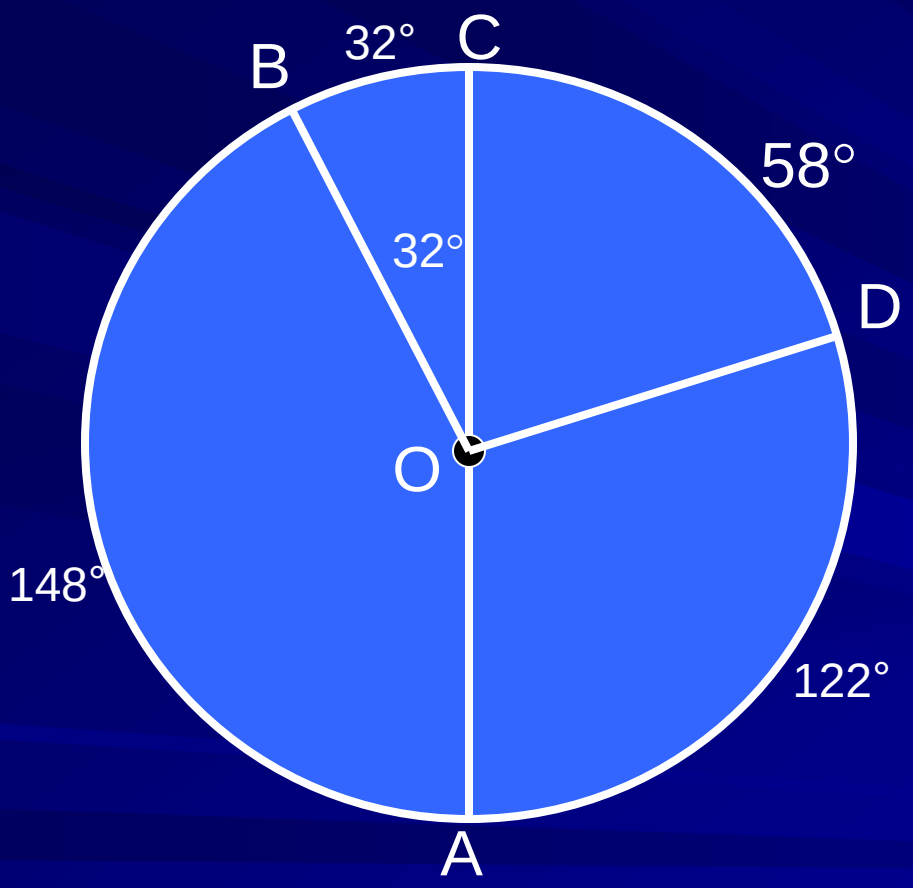
$m\angle CAD = (.08)(360)$

$m\angle CAD = 28.8°$

$m\angle DAE = 27\%$ of 360

$(.27)(360) = 97.2°$

Ex 1 : Finding the measures of Arcs



$m\widehat{BC}$ = m∠BOC = 32°

$m\widehat{DB}$ = $m\widehat{BC}$ + $m\widehat{CD}$

= 32° + 58° = 90°

$m\widehat{AD}$ = $m\widehat{ADC}$ – $m\widehat{CD}$

= 180° – 58° = 122°

$m\widehat{AB}$ = $m\widehat{ABC}$ – $m\widehat{BC}$

= 180° – 32° = 148°

Circumference

- Circumference – the distance around the circle.

$$C = d\pi = 2\pi r$$

Diameter

Radius

$\text{pi} \approx 3.1415$

Ex. 2: Find the circumference of the following circle.

$C = 2\pi r$

$= 2\pi(9\text{cm})$

$= 18\pi$ cm

≈ 56.549 cm

Ex. 3:

Got It? A car has a circular turning radius of 16.1 ft. The distance between the two front tires is 4.7 ft. How much farther does a tire on the outside of the turn travel than a tire on the inside?

radius of outer circle = 16.1

radius of inner circle = 16.1 − 4.7 = 11.4

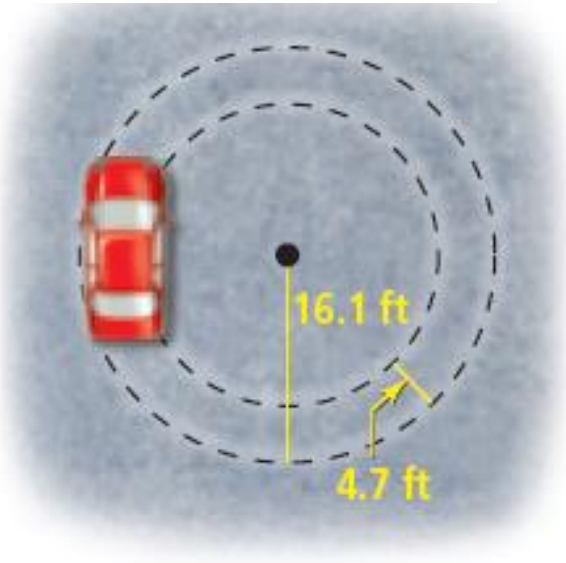
circumference of outer circle $= 2\pi r = 2\pi \cdot$ 16.1 $=$ 32.2 $\cdot \pi$

circumference of inner circle $= 2\pi r = 2\pi \cdot$ 11.4 $=$ 22.8 $\cdot \pi$

Find the differences in the two distances traveled. Use a calculator.

32.2 $\cdot \pi -$ 22.8 $\cdot \pi =$ 9.4 $\cdot \pi$

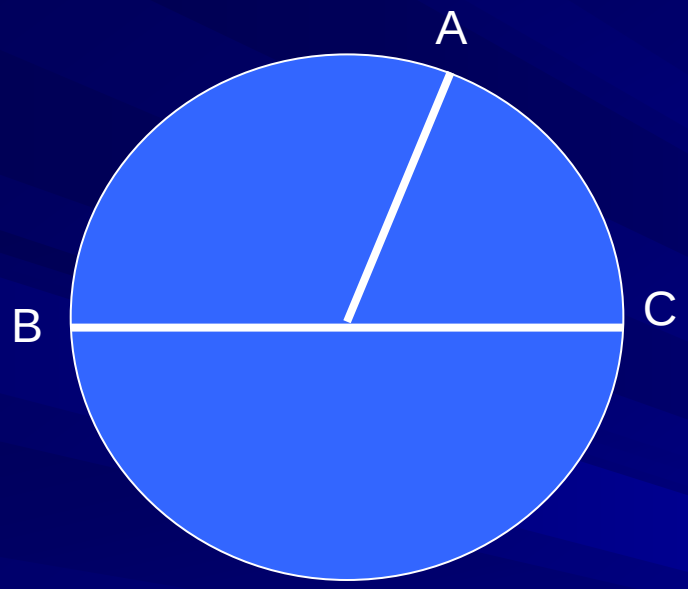
$\approx$ 29.53



A tire on the outside travels about 29.53 ft farther than a tire on the inside.

Arc Length

- The measure of an arc is in *degrees*.



- The length of an arc is a fraction of a circle's circumference. It is the length of a piece of string that would wrap around that part of the circle.

$$\text{Length of } \widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$$

Arc measure (in degrees)

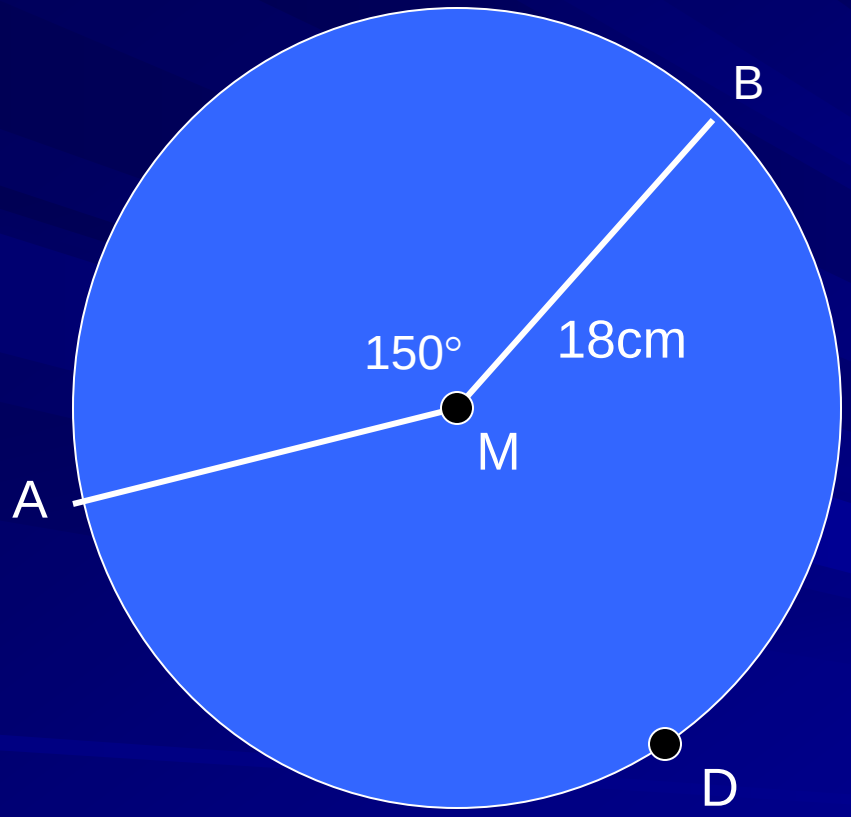
Circumference

Ex: An arc of 40° represents 40/360 or 1/9 of the circle.

We call this the “circle fraction” as it represents the portion of the circle in question.

Ex. 4:

Find the length of $\overset{\frown}{ADB}$ in ⊙M.



$m\overset{\frown}{ADB} = 210$

Length of $\overset{\frown}{ADB}$ = $m\overset{\frown}{ADB}$/360 • 2πr

Length of $\overset{\frown}{ADB}$ = (210/360) • 2(18) • π

Length of $\overset{\frown}{ADB}$ = 21 π cm

Length of $\overset{\frown}{ADB}$ ≈ 65.973 cm

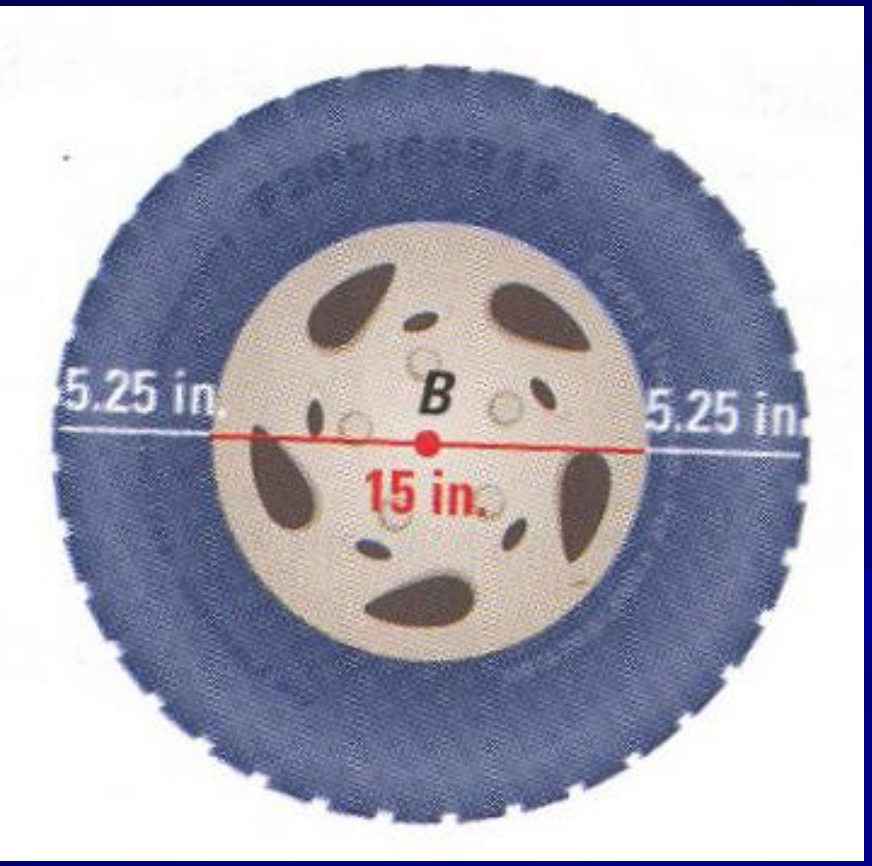
Ex. 5: Comparing Circumferences

- Tire Revolutions: Tires from two different automobiles are shown. How many revolutions does each tire make while traveling 100 feet? Round decimal answers to one decimal place.

Ex. 5: Comparing Circumferences

- Tire A has a radius of 7 + 5.1, or 12.1 inches.
- Its circumference is 24.2π, or about 76.027 inches.

Ex. 5: Comparing Circumferences



- Tire B has a radius of 7.5 + 5.25 or 12.75 inches.
- Its circumference is 25.5π, or about 80.111 inches.

Ex. 5: Comparing Circumferences

- Divide the distance traveled by the tire circumference to find the number of revolutions made. First: convert 100 feet to 1200 inches.

TIRE A: $\frac{100 \text{ ft.}}{76.027 \text{ in.}} = \frac{1200 \text{ in.}}{76.027 \text{ in.}}$

TIRE B: $\frac{100 \text{ ft.}}{80.111 \text{ in.}} = \frac{1200 \text{ in.}}{80.111 \text{ in.}}$

- Then divide the resulting numbers.

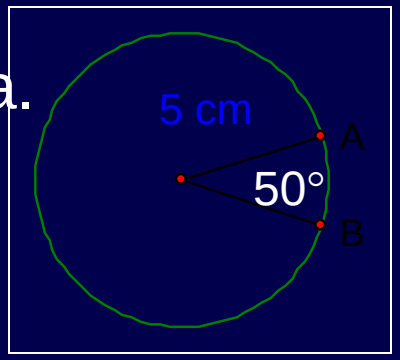
$\approx$ 15.784 revolutions

$\approx$ 14.979 revolutions

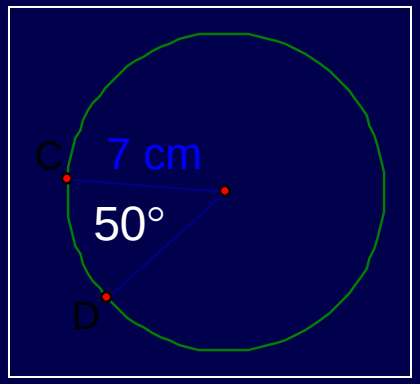
Ex. 6: Finding Arc Lengths

- Find the length of each arc.

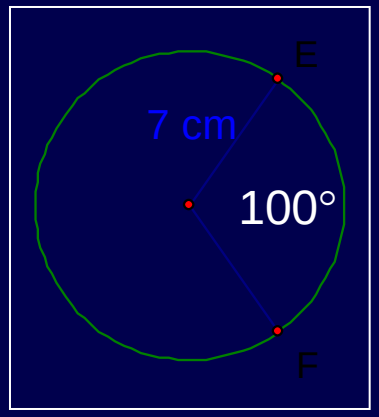
a.



b.



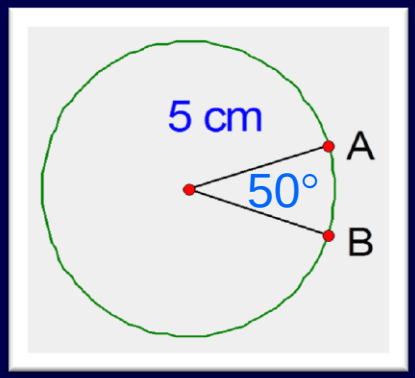
c.



Ex. 6: Finding Arc Lengths

- Find the length of each arc.

a.



Arc length of $\overset{\frown}{AB}$ = $\frac{\#\text{ of }^\circ}{360^\circ} \cdot 2\pi r$

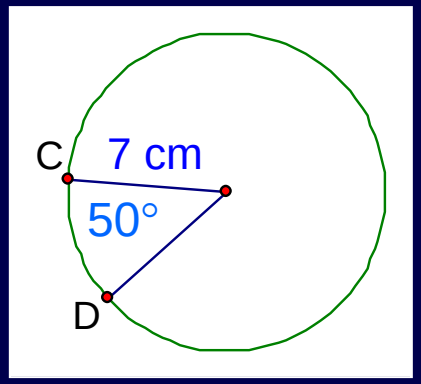
Arc length of $\overset{\frown}{AB}$ = $\frac{50^\circ}{360^\circ} \cdot 2\pi(5)$

$\approx$ 4.36 centimeters

Ex. 6: Finding Arc Lengths

- Find the length of each arc.

b.



Arc length of $\overset{\frown}{AB}$ = $\frac{\#\text{ of }^\circ}{360^\circ} \cdot 2\pi r$

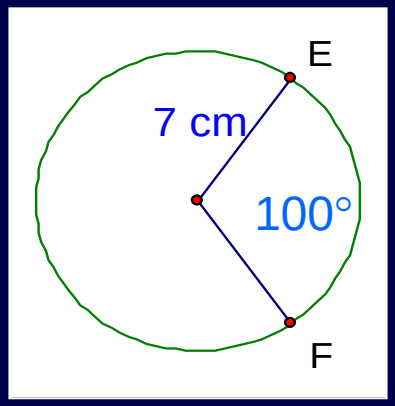
Arc length of $\overset{\frown}{AB}$ = $\frac{50^\circ}{360^\circ} \cdot 2\pi(7)$

$\approx$ 6.11 centimeters

Ex. 6: Finding Arc Lengths

- Find the length of each arc.

c.



Arc length of $\widehat{AB}$ = $\frac{\#\text{ of }^\circ}{360^\circ} \cdot 2\pi r$

Arc length of $\widehat{AB}$ = $\frac{100^\circ}{360^\circ} \cdot 2\pi(7)$

$\approx$ 12.22 centimeters

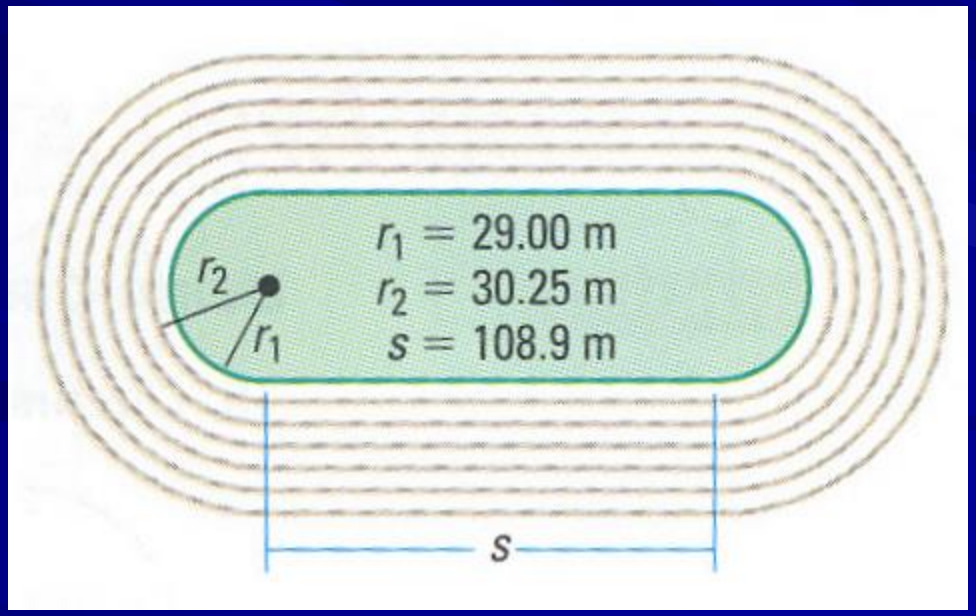
In parts (a) and (b) in Example 6, note that the arcs have the same measure but different lengths because the circumferences of the circles are not equal. In part (c) the length is twice (b) because the angle measure is double that of (b).

Ex. 7: Finding Arc Length

- Track. The track shown has six lanes. Each lane is 1.25 meters wide. There is 180° arc at the end of each track. The radii for the arcs in the first two lanes are given.

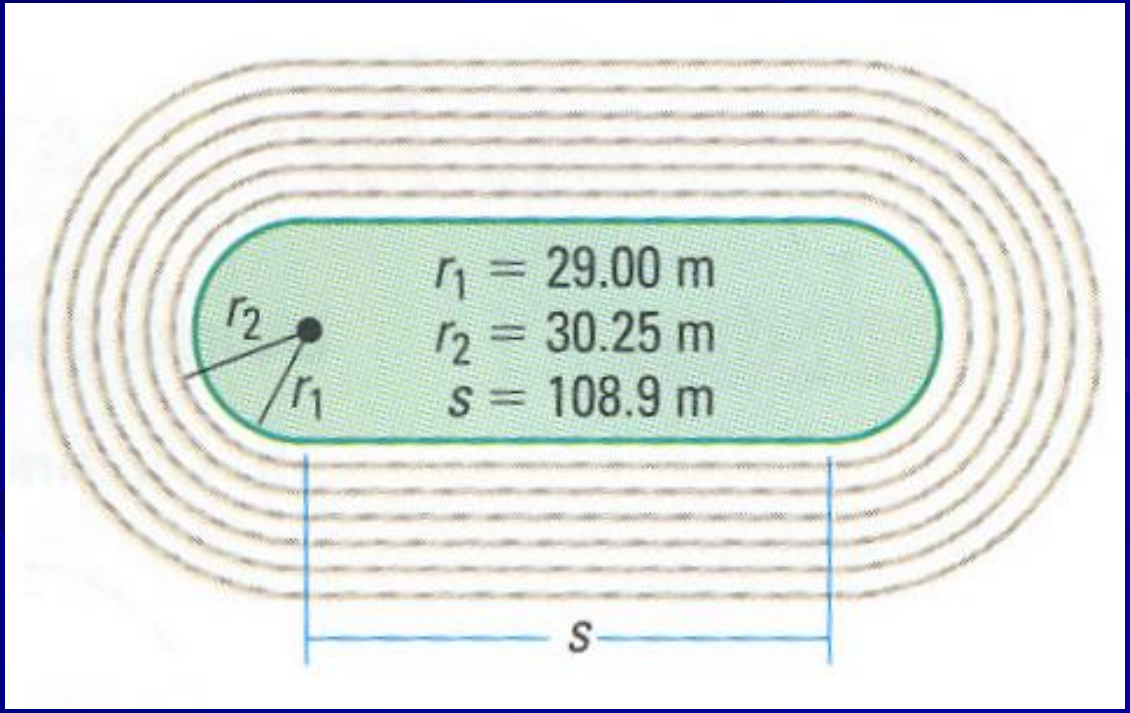
a. Find the distance around Lane 1.

b. Find the distance around Lane 2.



Ex. 7: Finding Arc Length

- The track is made up of two semicircles and two straight sections with length s. To find the total distance around each lane, find the sum of the lengths of each part. Round decimal answers to one decimal place.



Lane 1

- Distance = $2s + 2\pi r_1$

  $= 2(108.9) + 2\pi(29.00)$

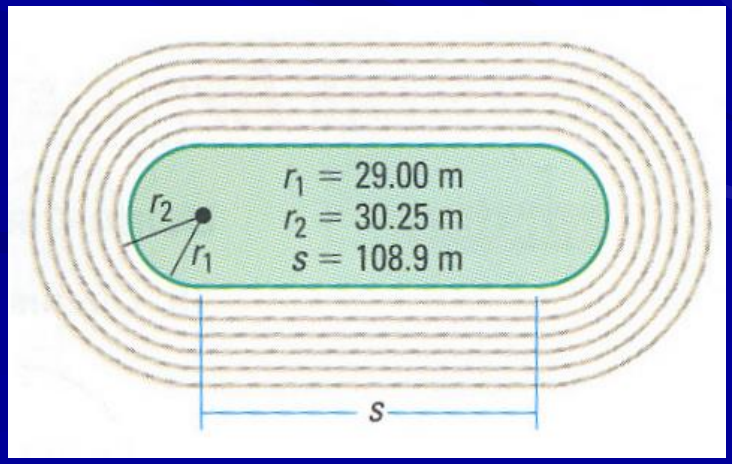
  ≈ 400.0 meters

Lane 2

- Distance = $2s + 2\pi r_2$

  $= 2(108.9) + 2\pi(30.25)$

  ≈ 407.9 meters



Exit Slip

- In words, explain the difference between arc measure and arc length. (2-3 sentences)

- In words, explain how to find arc length of a circle. (Include a formula!)